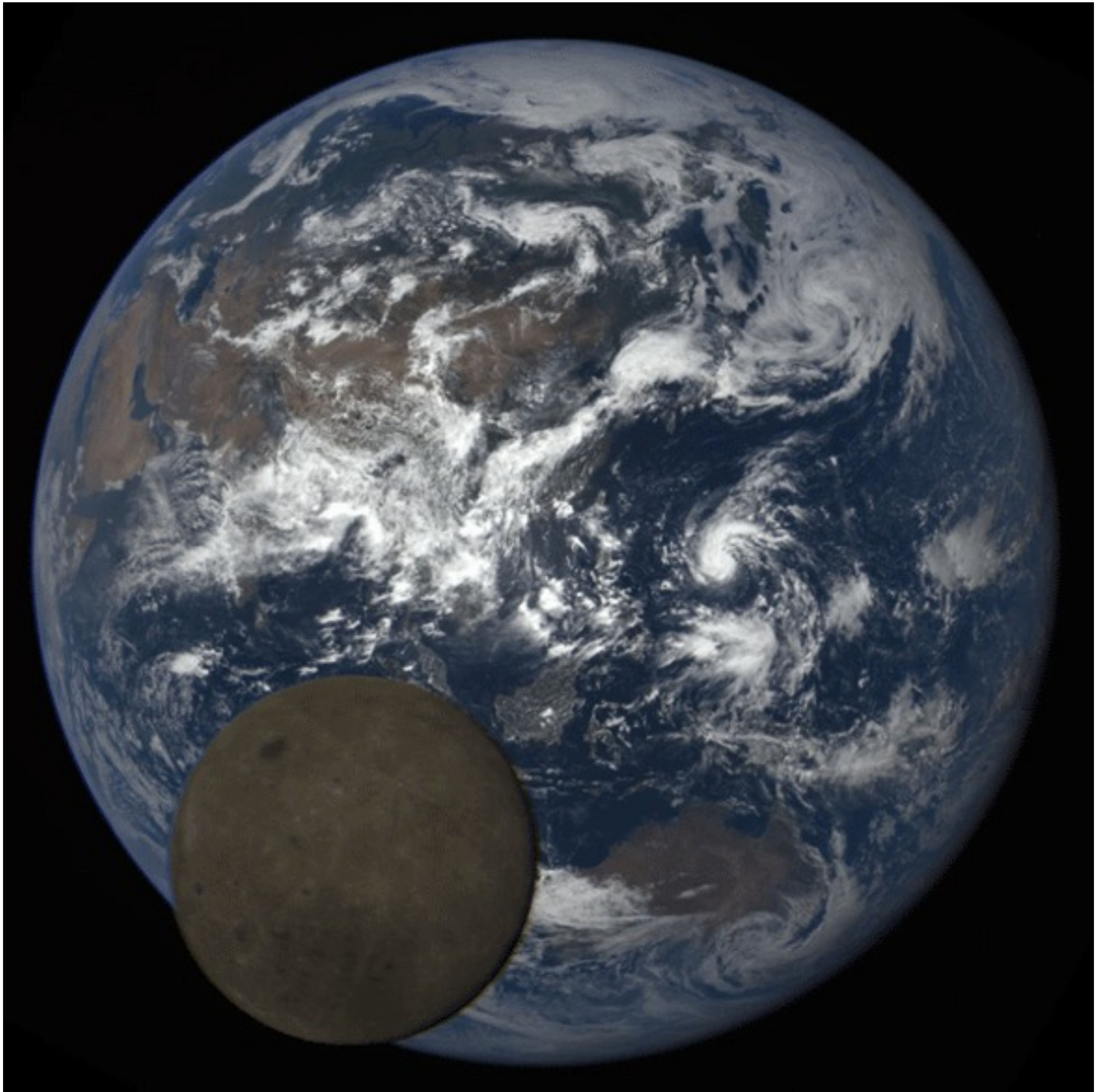
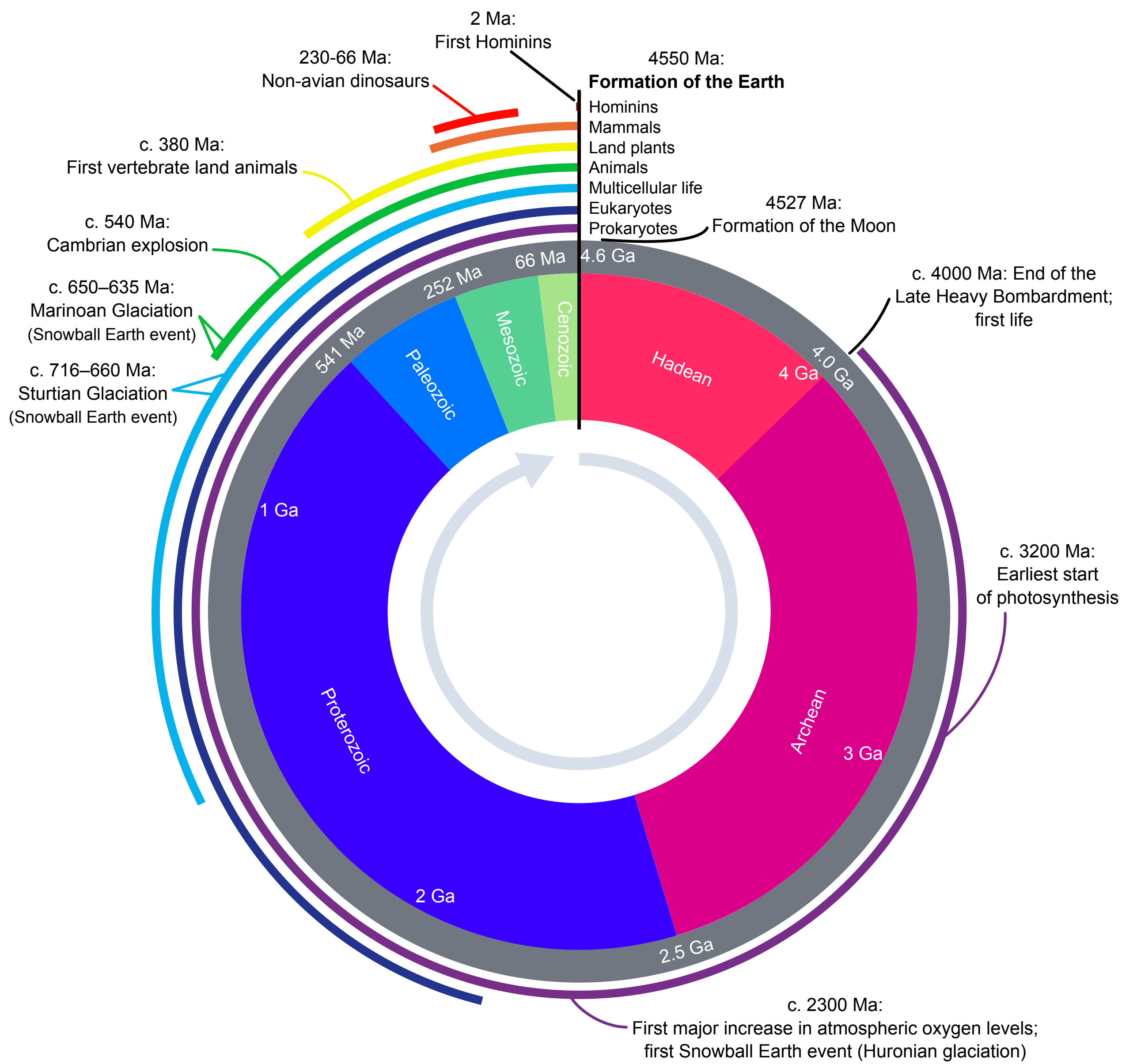
Thermal Distribution in Earth's Interior

ME 531 Graduate Conduction Heat Transfer
at the University of Washington
Mechanical Engineering Dept.

Professor Ann Mescher



4.54 Billion Year Earth History



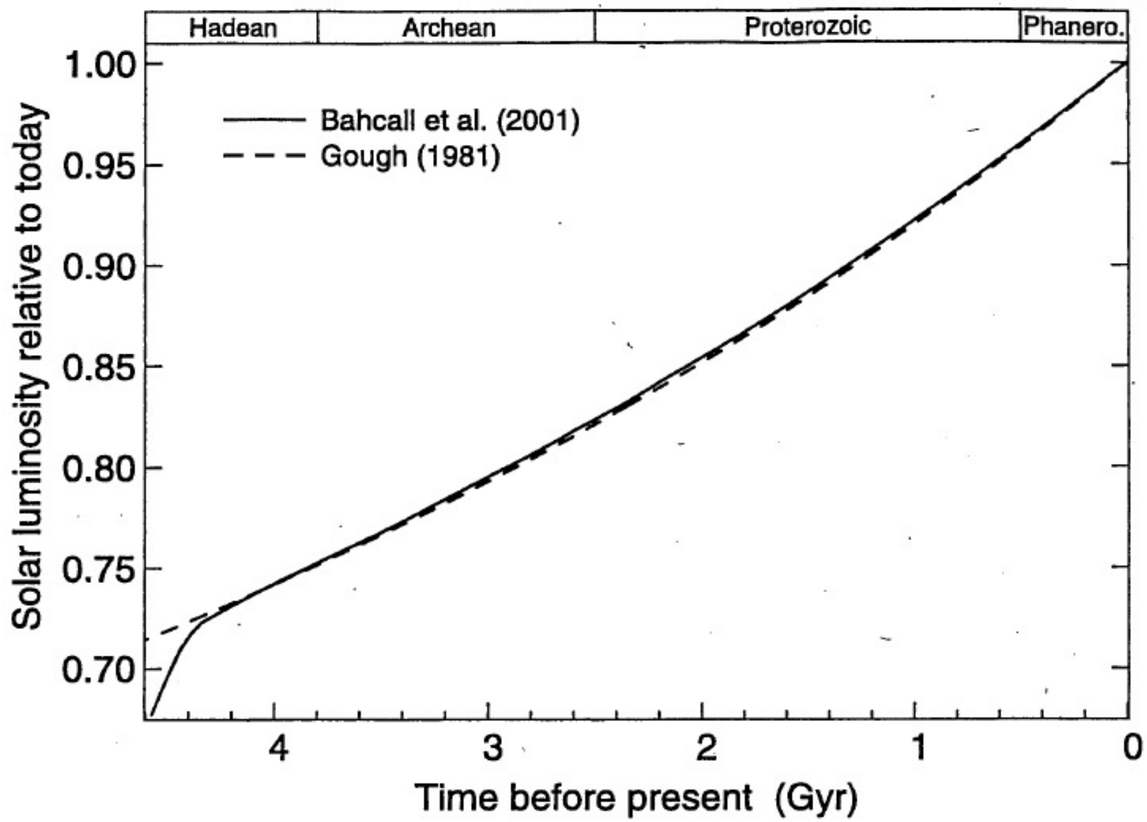


Figure 1. Evolution of solar luminosity over the four geologic eons for the standard solar model described in *Bahcall et al.* [2001] (solid line) and according to the approximation formula [*Gough*, 1981] (dashed line) given in equation (1).

Consensus on Evolution of Solar Luminosity

"The gradual increase in luminosity during the core hydrogen burning phase of evolution of a star is an inevitable consequence of Newtonian physics and the functional dependence of the thermonuclear reaction rates on density, temperature and composition." [Gough, 1981, p. 28]

Liquid Water Evidence in Archean Eon

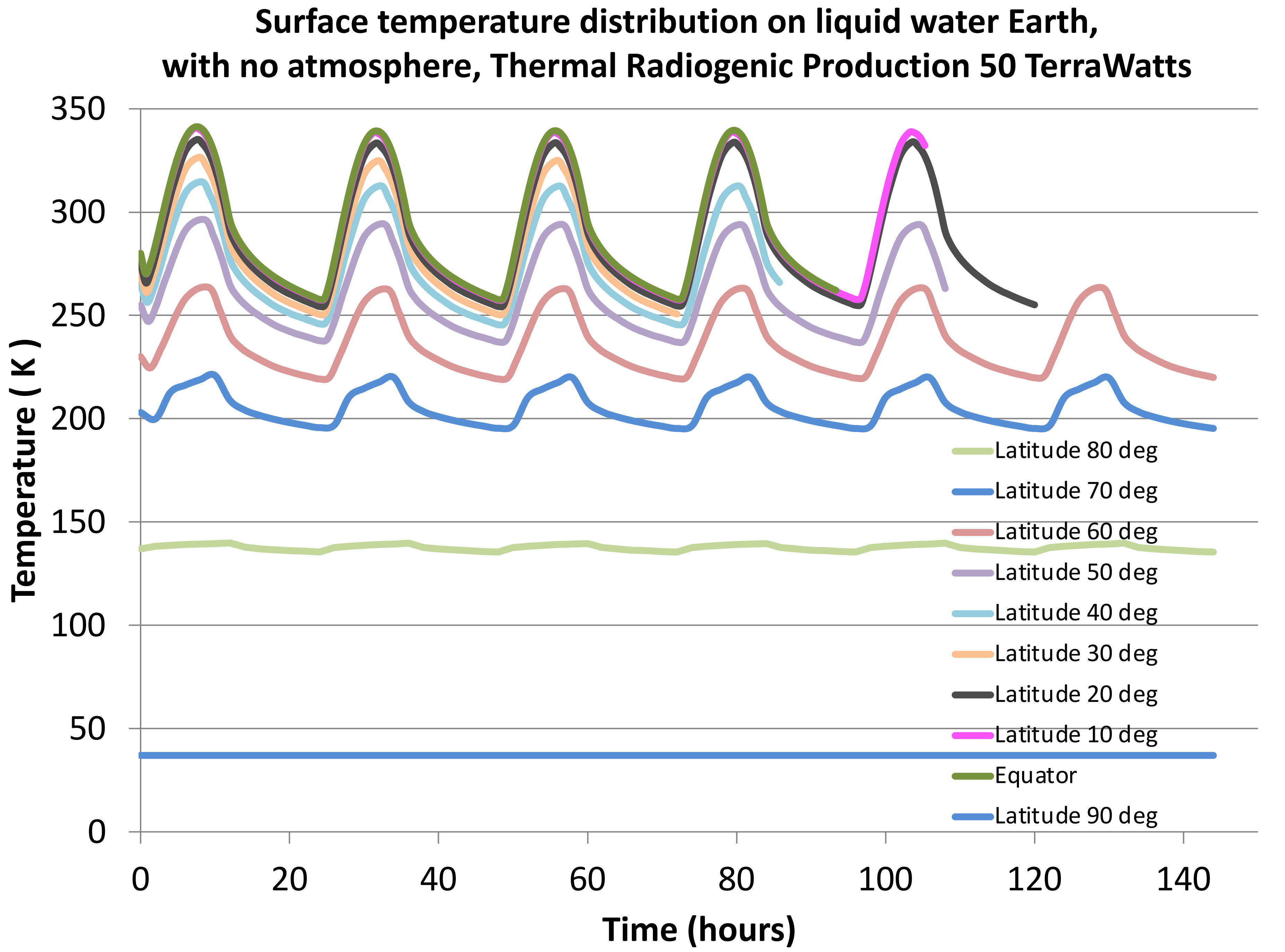
- "Telltale signs of liquid water include pillow lavas that are formed when lava extrudes under water, ripple marks resulting from sediment deposition under the influence of waves, and mud cracks."
- " . . evidence for microbial life in the Archean derived from microfossils or stromatolites / microbial mats in rocks of ages between 2.5 and 3.5 Gyr [*Barghoorn and Schopf*, 1966; *Altermann and Kazmierczak*, 2003; *Schopf*, 2006]"
- " . . no evidence for widespread glaciations during the entire Archean [Lowe, 1980; Walker, 1982; Walker et al, 1983; Fowler et al, 2002; Eriksson et al, 2004; Benn et al, 2006]"

“Paradox” from Global Energy Balance

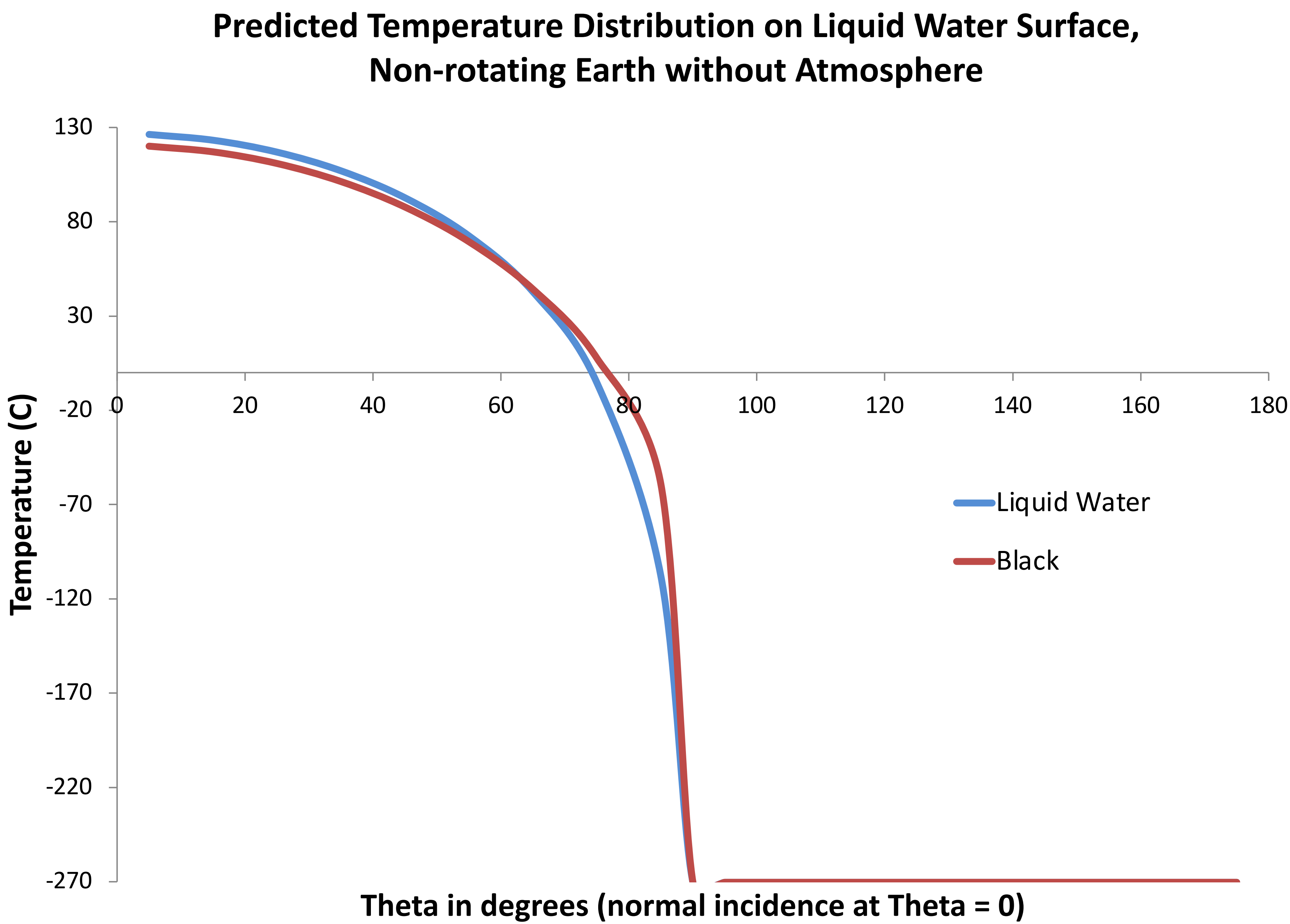
- Earth’s global ***average*** surface temperature is ~ 288 K, governed by current normal solar incidence (1360 W/m^2) at the top of the atmosphere.
- ***Without*** thermal effects of the atmosphere, Earth’s global average surface temperature would instead be ~ 278 K. (“Thermal blanket” adds surface average ~ 10 K.)
- Without atmospheric “thermal blanketing,” Earth’s global ***average*** surface temperature would have been ~ 259K, at 75% of current solar luminosity.
- At 80% of current solar luminosity, Earth’s global ***average*** surface temperature would have been ~ 263K, still well below H_2O freezing at standard atmospheric pressure.

Why Was Archean Earth Not Frozen?

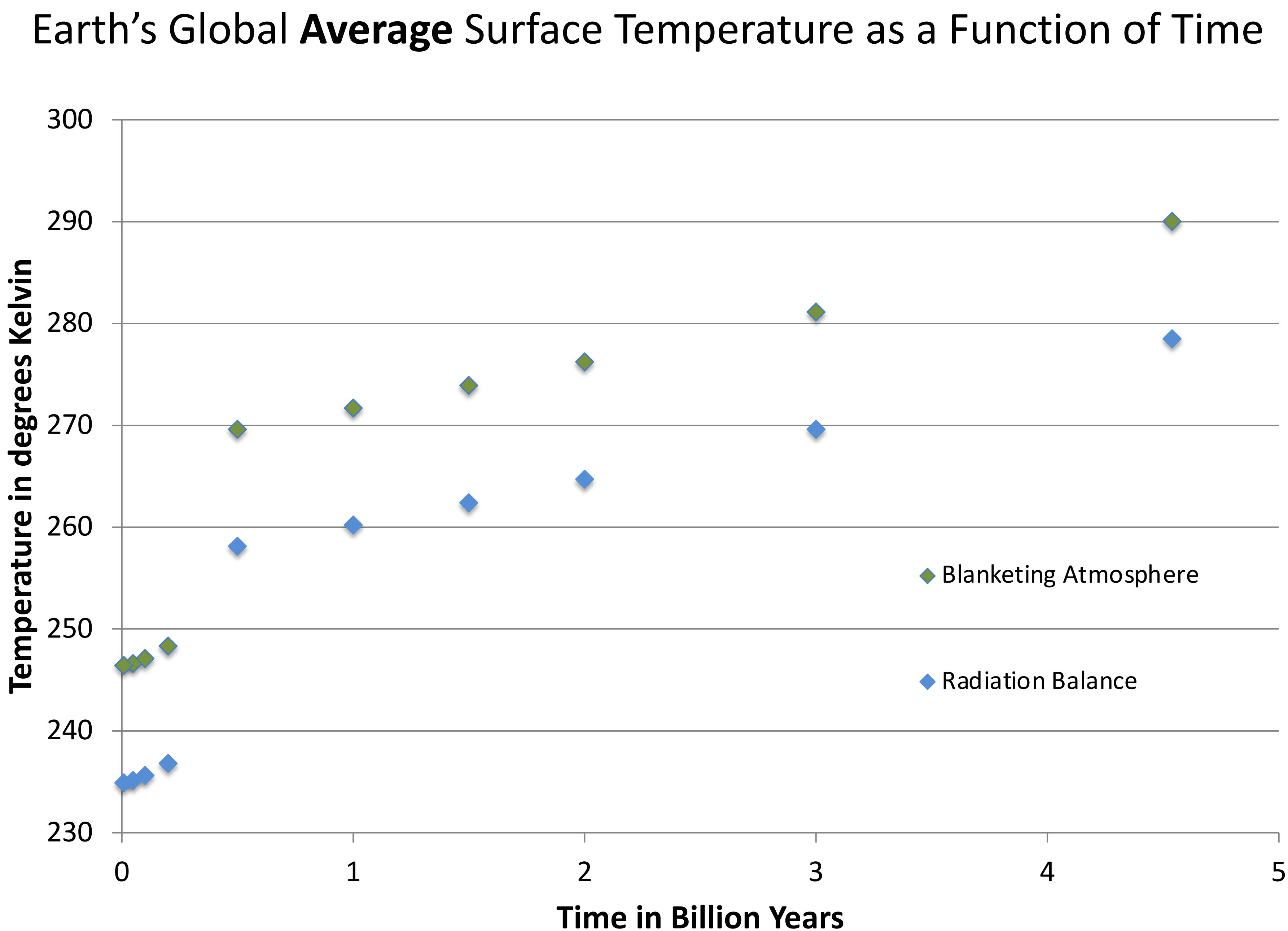
- ✓ "Observations of other cool stars show that they lose most of their mass during first 0.1 Gyr. Most importantly, the observed solar analogs exhibit considerably lower cumulative mass-loss rates than required to offset the low luminosity of the early Sun [*Minton and Malhotra*, 2007]."
- ✓ Enhanced green house effects of Ammonia, Methane, Carbon Dioxide, other greenhouse gases checked.
- ✓ Cloud effects: " . . it appears unlikely that any cloud effect alone can resolve the faint young Sun problem . ."
- ✓ Rotation and Obliquity effects checked.
- ✓ Ocean Salinity and Tide effects checked.



Surface temperature distribution on liquid water Earth,
with no atmosphere, Thermal Radiogenic Production 50 TerraWatts
Temperature (K)
Time (hours)
0
50
100
150
200
250
300
350
0
20
40
60
80
100
120
140
Latitude 80 deg
Latitude 70 deg
Latitude 60 deg
Latitude 50 deg
Latitude 40 deg
Latitude 30 deg
Latitude 20 deg
Latitude 10 deg
Equator
Latitude 90 deg

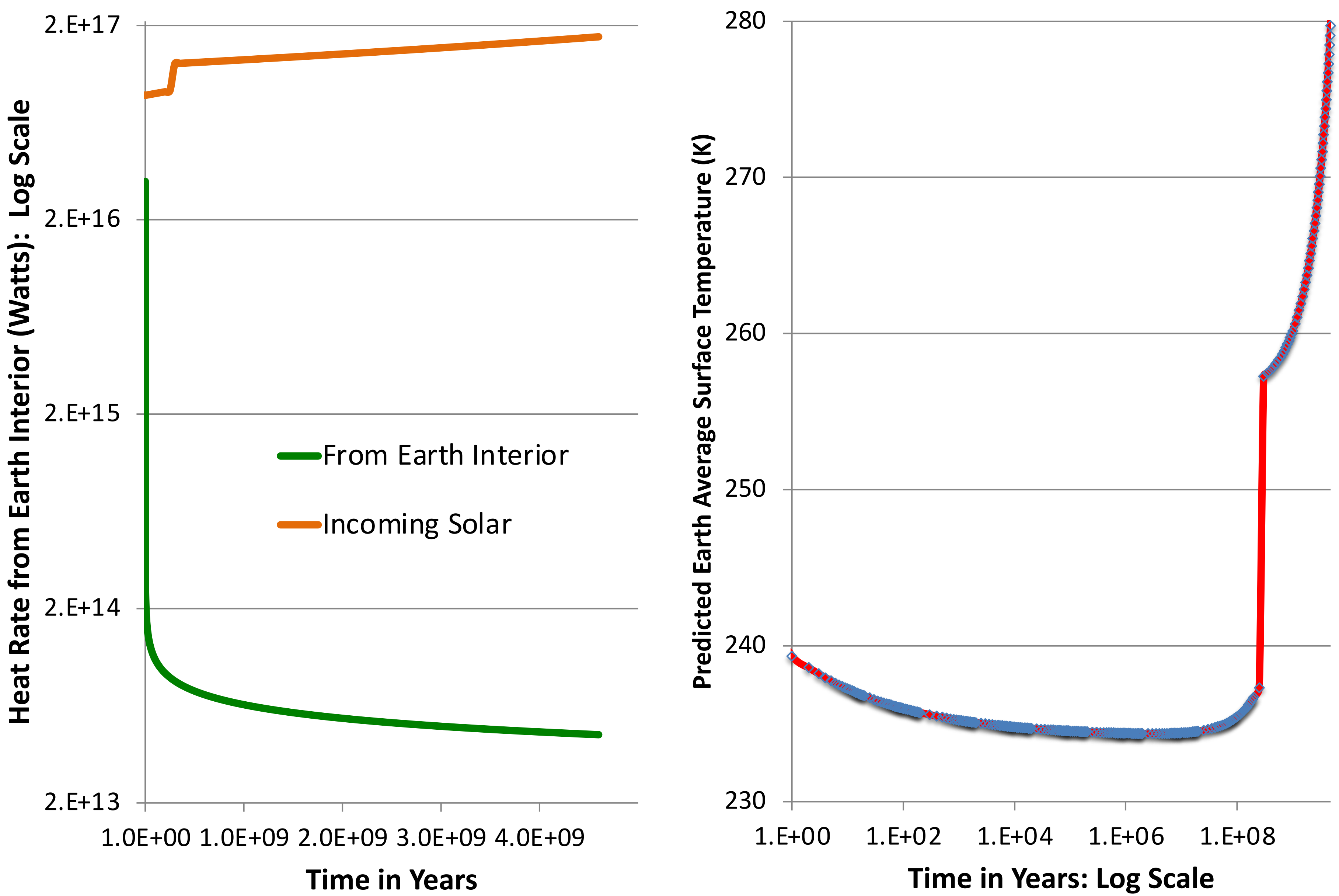


Predicted Temperature Distribution on Liquid Water Surface,
Non-rotating Earth without Atmosphere
Temperature (C)
130
80
30
-20
-70
-120
-170
-220
-270
0
20
40
60
80
100
120
140
160
180
Liquid Water
Black
Theta in degrees (normal incidence at Theta = 0)



Earth's Global **Average** Surface Temperature as a Function of Time
300
290
280
270
260
250
240
230
Temperature in degrees Kelvin
0
1
2
3
4
5
Time in Billion Years
Blanketing Atmosphere
Radiation Balance

With Early Solar Correction

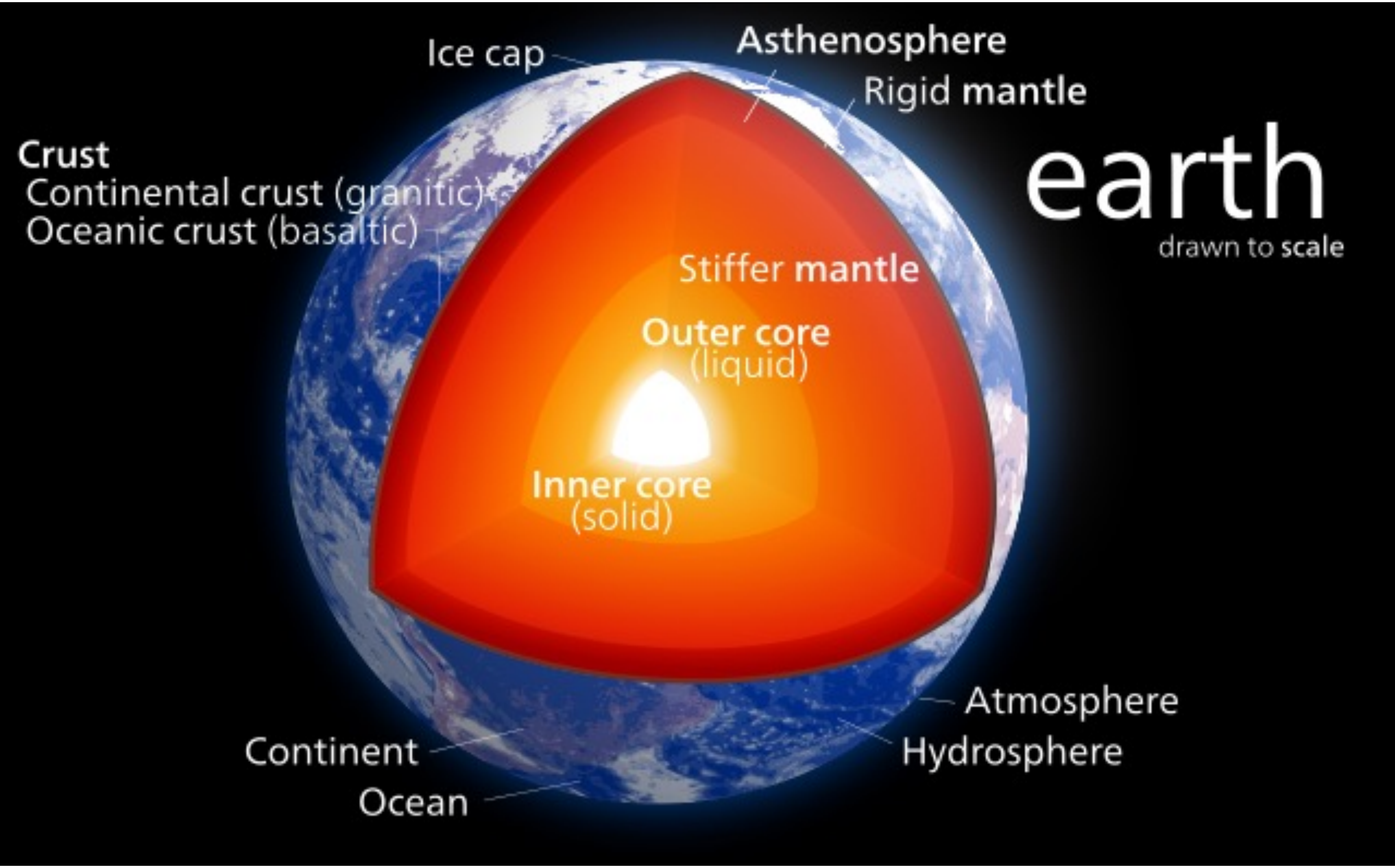


Intermediate (Important) Conclusions

For the purpose of modeling Earth's interior thermal distribution, it is reasonable to treat its surface temperature $T_{surface}$ as "constant" over a time period of 0.30 Billion to 4.54 Billion Years.

$T_{surface} \sim 280\ K$

Earth's average surface temperature is indeed affected by its atmosphere, but far and away the most dominant influence is solar flux.


Ice cap
Asthenosphere
Rigid mantle
earth
drawn to scale
Crust
Continental crust (granitic)
Oceanic crust (basaltic)
Stiffer mantle
Outer core
(liquid)
Inner core
(solid)
Atmosphere
Hydrosphere
Continent
Ocean

Earth's Interior Challenges

Thermal conductivities, densities and specific heats vary spatially with material composition and as a function of temperature.

+Thermal diffusivities vary far less

Radiogenic heat production is a function of spatial coordinates and time.

+Current source estimates ~20-25 TW likely to be close; estimates of historic radiogenic production reasonable.

Total heat rate from interior based on 38,000 measurements, reported as 47 +/-2 TeraWatts.

Why attempt analytical modeling?
(Rather than proceeding full force with simulation?)

Disparate physical scales:

- Average Earth radius ~ 6371 km
- Conduction at kinetic molecular collision scale

Disparate time scales:

- Earth age ~ 4.54 Billion Years
- Simulate conduction with time steps ~seconds

Conduction within solid spherical Earth, where r = radius, θ = latitudinal angle, and φ = circumferential angle. The outer radius, R = 6371 km. The current radiogenic production, $u^{'''}\forall \approx$ 20 TW decreasing from $u^{'''}\forall \approx$100TW @ accretion.

For uniform conductivity k:
$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial T}{\partial \theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \varphi^2} + \frac{u^{'''}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$.

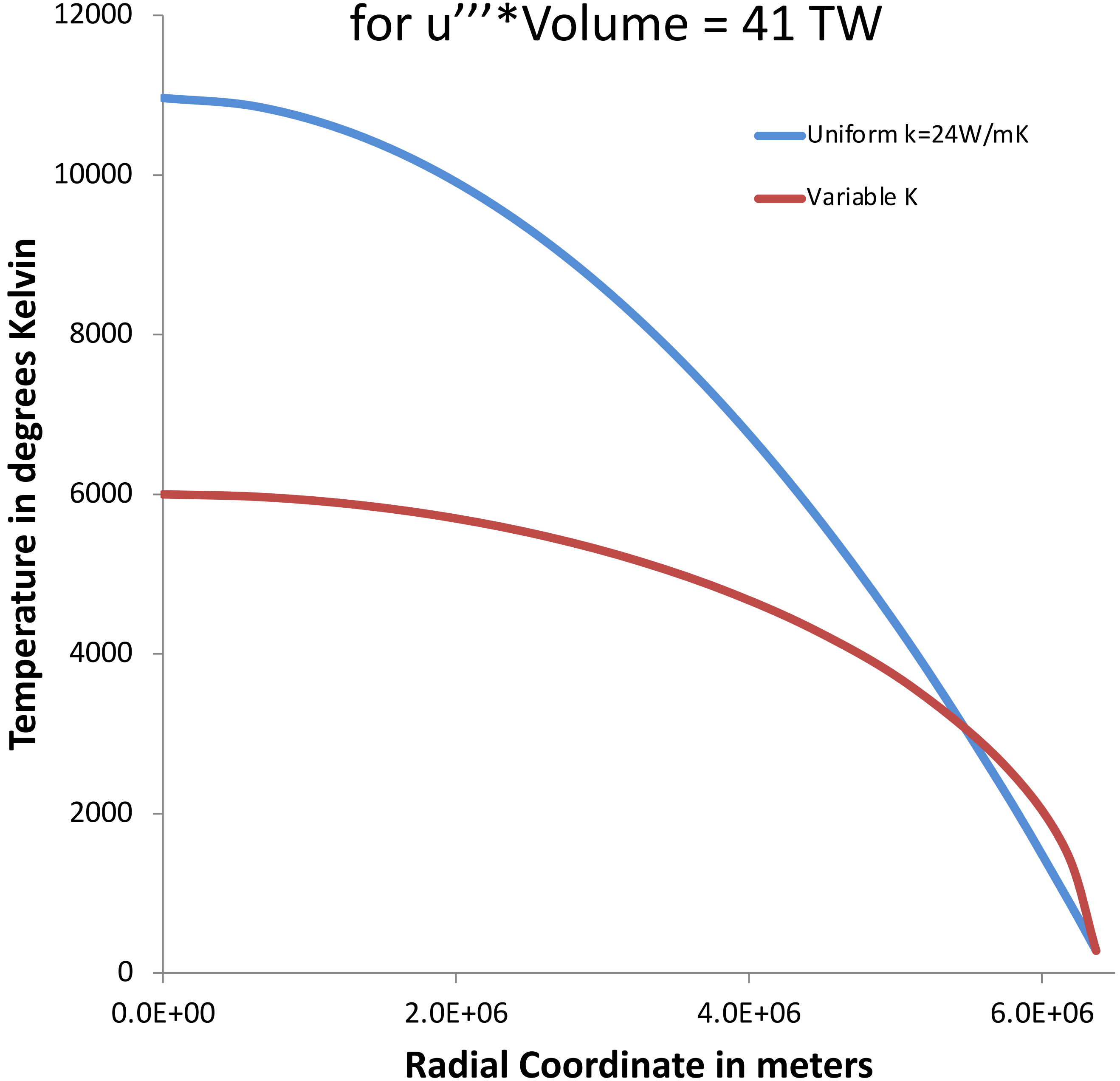
For variable thermal conductivity k:
$\frac{1}{r^2}\frac{\partial}{\partial r}(kr^2\frac{\partial T}{\partial r}) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(k\sin\theta\frac{\partial T}{\partial \theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \varphi}(k\frac{\partial T}{\partial \varphi}) + u^{'''} = \rho c\frac{\partial T}{\partial t}$.

Boundary conditions? $T|_{\varphi} = T|_{\varphi+2\pi}$, and $\frac{\partial T}{\partial \varphi}|_{\varphi} = \frac{\partial T}{\partial \varphi}|_{\varphi+2\pi}$.

Likewise in θ, continuity in temperature and thermal gradient $\frac{\partial T}{\partial \theta}$.
Largest thermal changes in radial r.
Neglecting $\frac{\partial T}{\partial \varphi}$ and $\frac{\partial T}{\partial \theta}$, the boundary conditions in r:
$\frac{\partial T}{\partial r}|_{r=0} = 0$, and $T|_{r=R} = T_{\text{surface}} \simeq 280K$.

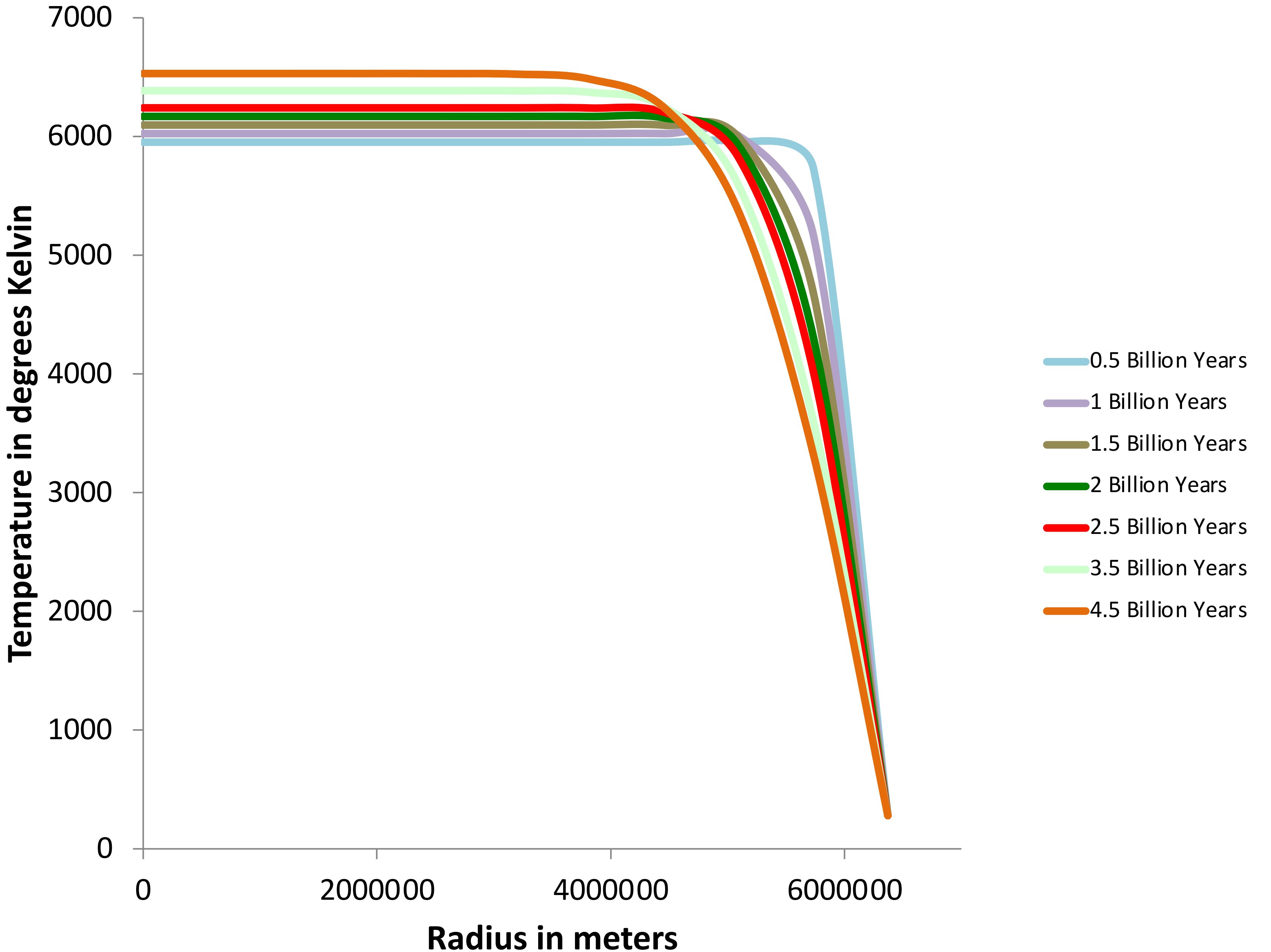
Solve $\frac{1}{r^2}\frac{\partial}{\partial r}(kr^2\frac{\partial T}{\partial r}) + u^{'''} = \rho c\frac{\partial T}{\partial t}$.

Earth's "steady" Interior Temperatures for u'''*Volume = 41 TW

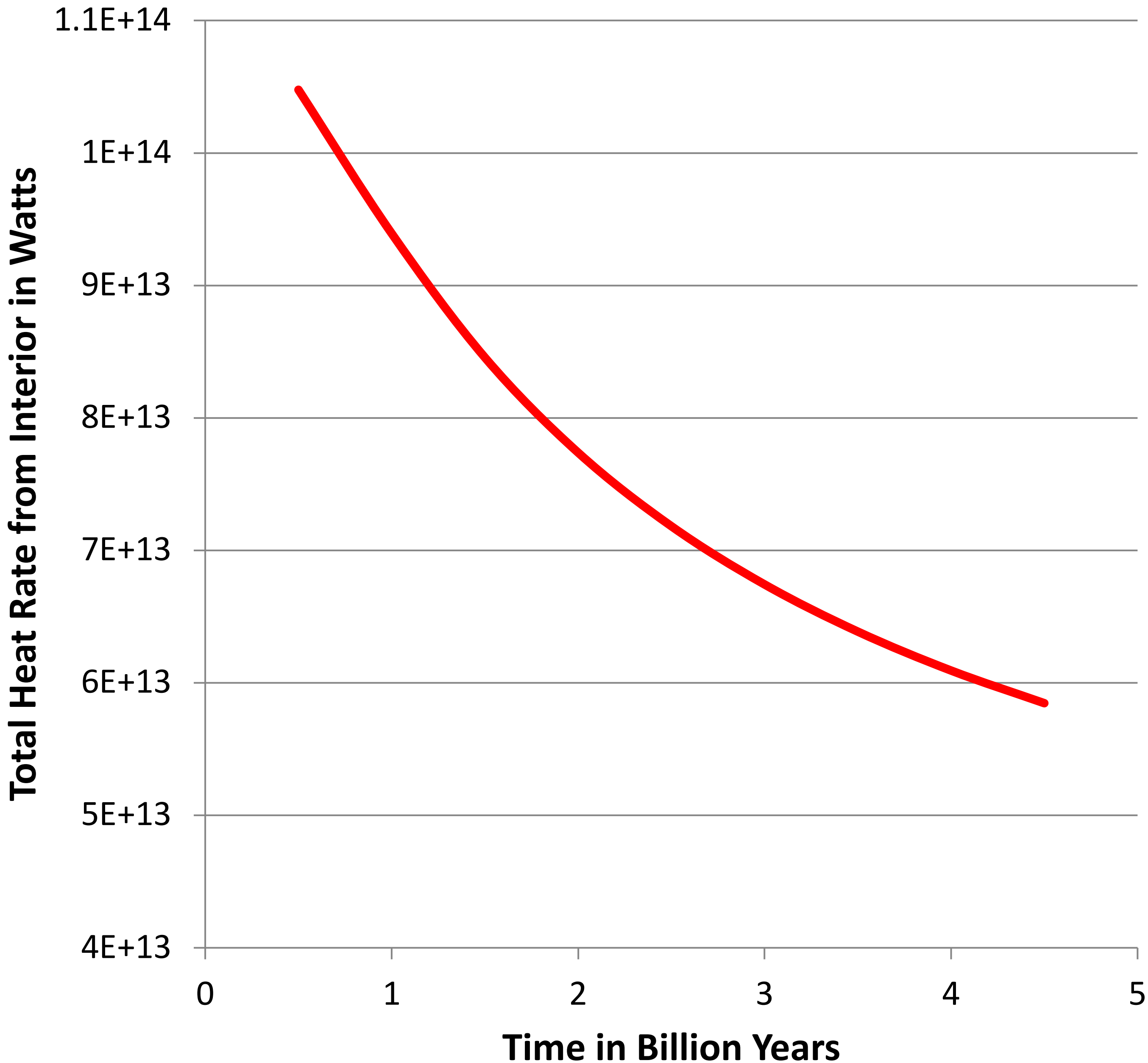


Is the Earth close at all to steady state?

The answer is NO: Prediction for k = 24 W/(m K), u'''V = 41 TW

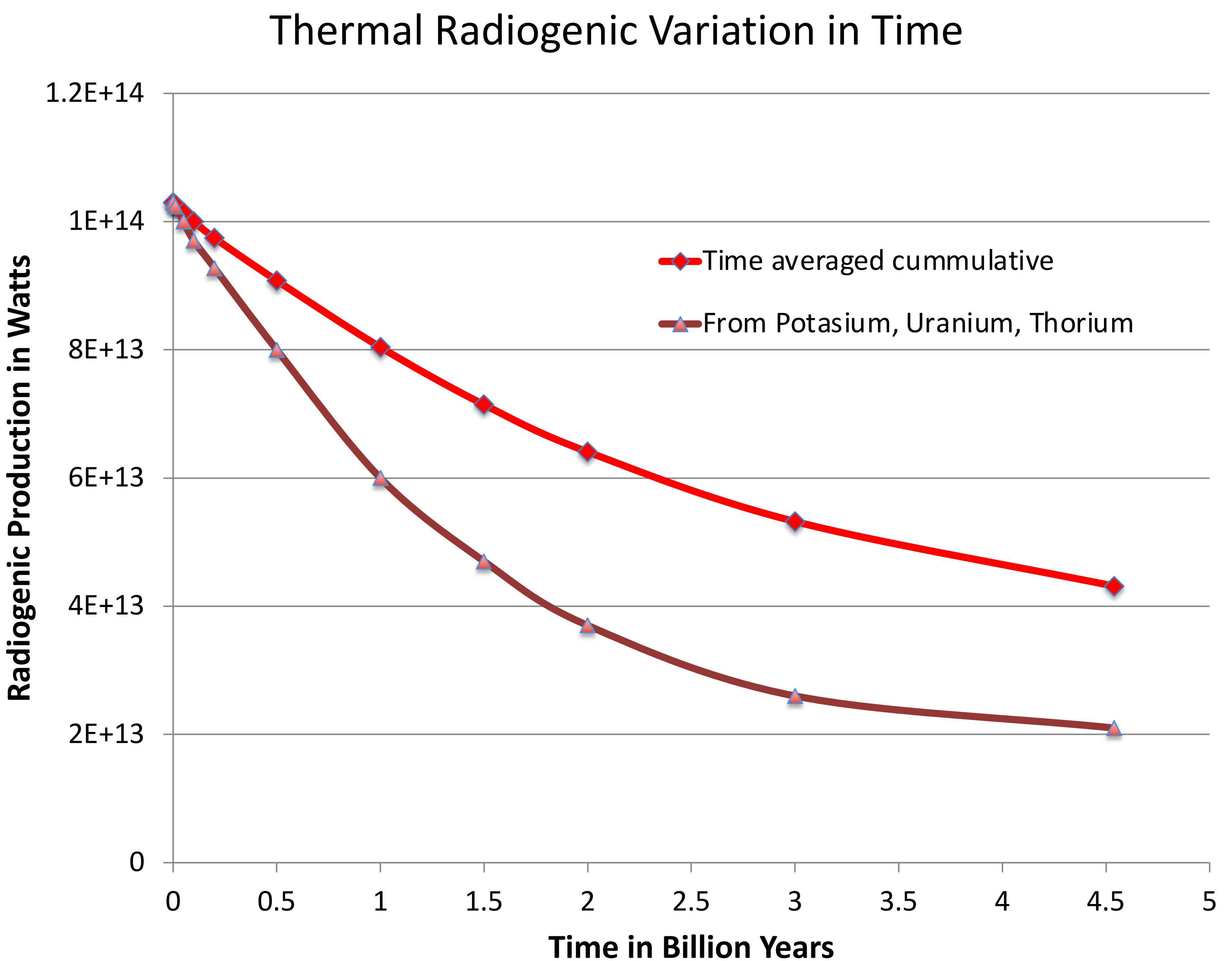


Total Heat Rate from Interior, k = 24 W/(m K), u’’’V = 41 TW

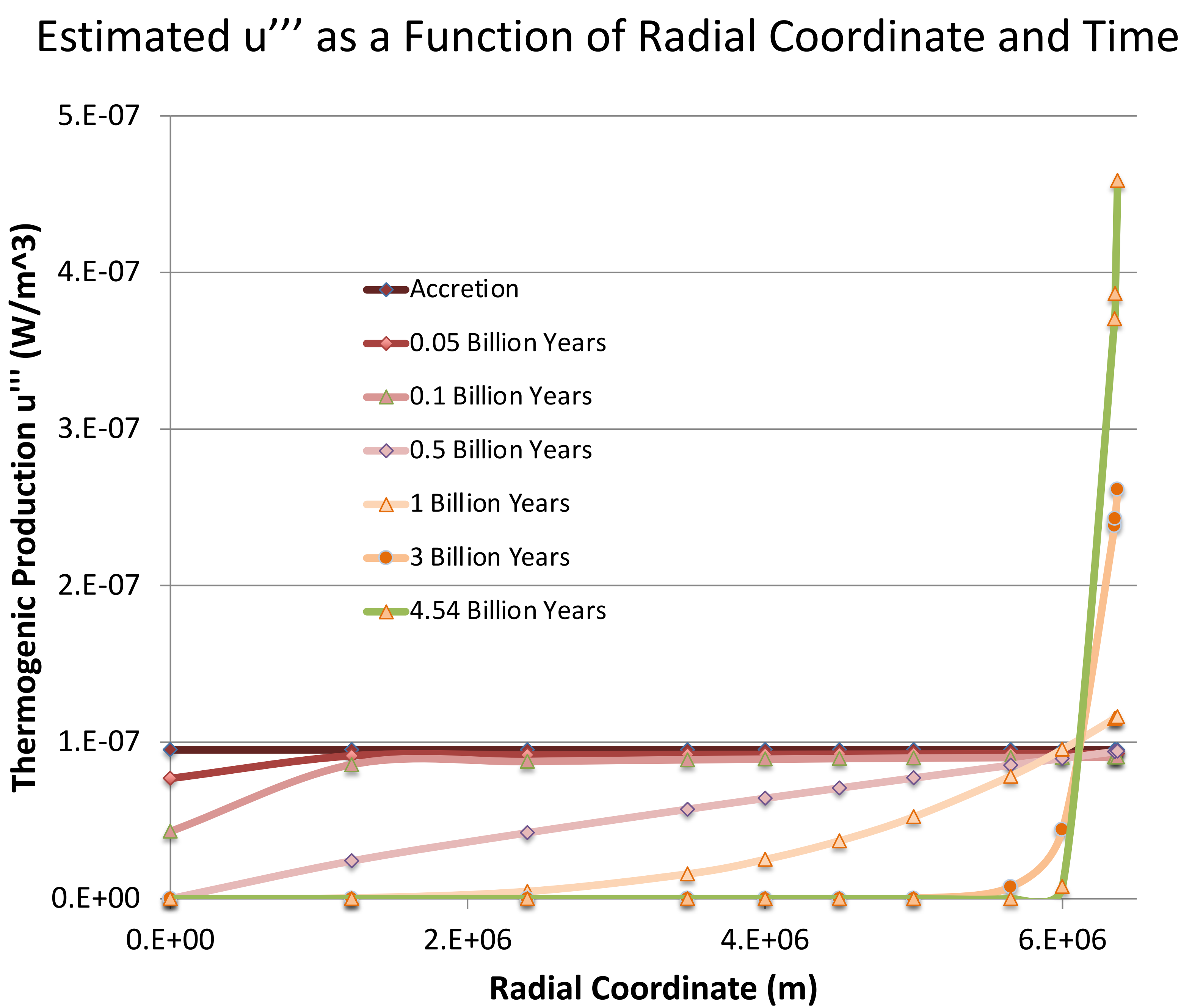


Two most easily identified problems:

- The Earth's interior thermal conductivity is not uniform!
- The Earth's thermogenic production is spatially and temporally variable!

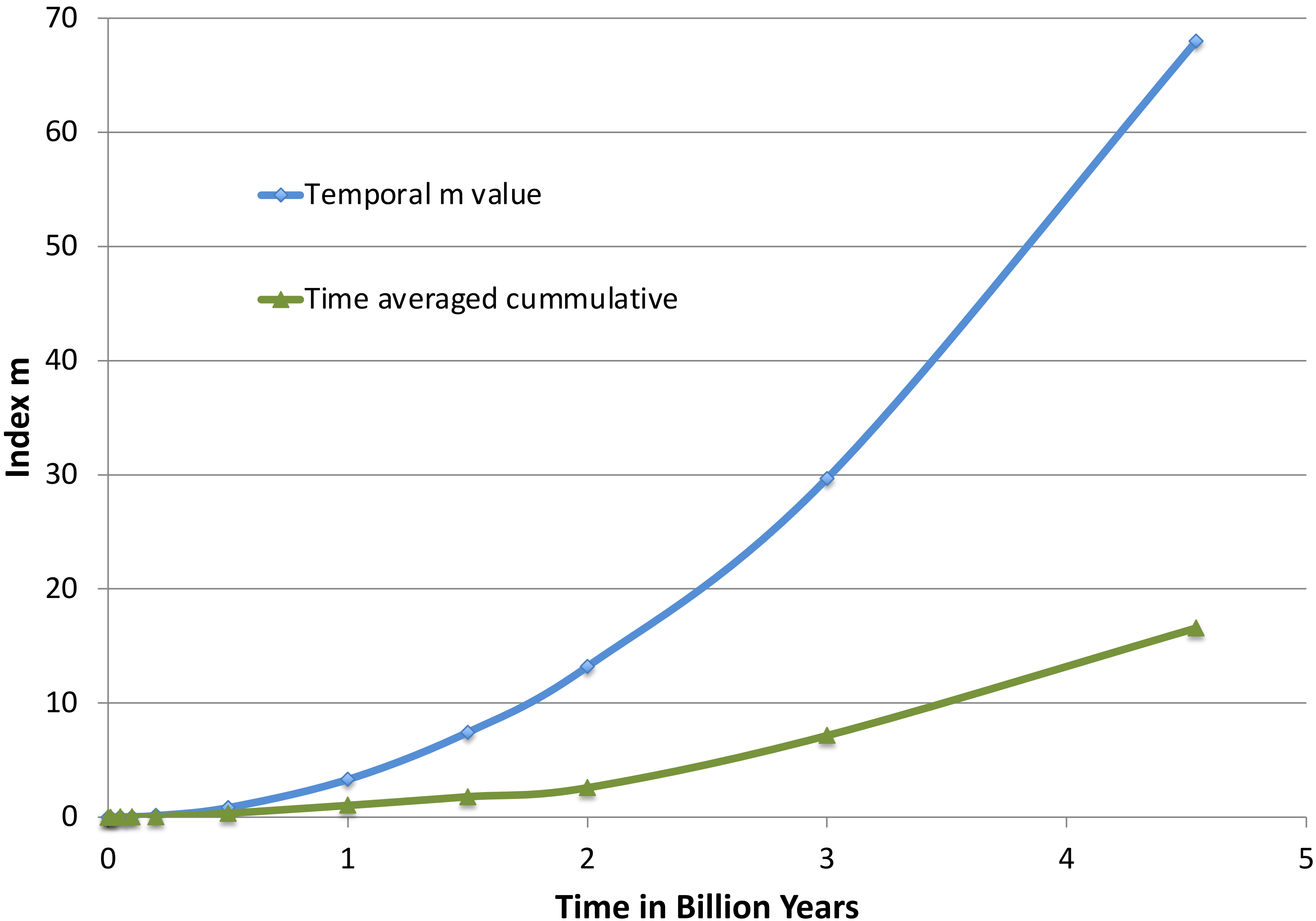


Thermal Radiogenic Variation in Time
Radiogenic Production in Watts
1.2E+14
1E+14
8E+13
6E+13
4E+13
2E+13
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time in Billion Years
Time averaged cummulative
From Potasium, Uranium, Thorium



Estimated u''' as a Function of Radial Coordinate and Time
Thermogenic Production u''' (W/m^3)
5.E-07
4.E-07
3.E-07
2.E-07
1.E-07
0.E+00
0.E+00
2.E+06
4.E+06
6.E+06
Radial Coordinate (m)
Accretion
0.05 Billion Years
0.1 Billion Years
0.5 Billion Years
1 Billion Years
3 Billion Years
4.54 Billion Years

Varying index m: $3u''' = 3u_s''' (r/R)^m = (3+m)u_c'''(r/R)^m$



With uniform conductivity k: $T(r,t) = T_{\text{surface}} + \frac{u'''}{6k}(R^2 - r^2)$
$+ \sum_{n=1}^{\infty}\{\frac{2R(-1)^n}{n\pi}[\frac{u''' R^2}{k(n\pi)^2} - (T_i - T_{\text{surface}})]\frac{\sin(\frac{n\pi r}{R})}{r}\exp(\frac{-k(n\pi)^2}{\rho c R^2}t)\}$,
for initial uniform temperature $T(r, t = 0) = T_i$.

For variable thermal conductivity $k = k_s \frac{T}{T_s}$:
$T(r,t) = \sqrt{T_s^2 + T_s \frac{u'''}{3k_s}(R^2 - r^2)} + \sqrt{2\sum_{n=1}^{\infty} T_s B_n \frac{\sin(\frac{n\pi r}{R})}{r}\exp(-\alpha(\frac{n\pi}{R})^2 t)}$,
where thermal diffusivity $\alpha = \frac{k}{\rho c}$ is constant, and where:
$T_s B_n = \frac{T_s R^3 u'''(-1)^n}{3k_s n\pi}(1 - \frac{6}{(n\pi)^2}) - \frac{R(-1)^n}{n\pi}(T_i^2 + T_s^2 + T_s R^2 \frac{u'''}{3k_s})$
$-\frac{2T_i}{R}\int_o^R r\sin(\frac{n\pi r}{R})\sqrt{T_s^2 + T_s \frac{u'''}{3k_s}(R^2 - r^2)}dr$.

Analytical solution for $k = k_s(\frac{T}{T_s})^x$, and $u''' = u_s'''(\frac{r}{R})^m$? Yes!

"Steady state" solution $T(r) = [T_s^{x+1} + \frac{u_c''' R^2(x+1)T_s^x}{3k_s(m+2)}(1 - (\frac{r}{R})^{2+m})]^{\frac{1}{x+1}}$,
where $u_c''' = u_s''' \frac{3}{3+m}$, and $u'''|_{r=R} = u_s'''$.

Transient 1-D solution for $k = k_s(\frac{T}{T_s})^x$ and $u''' = u_s'''(\frac{r}{R})^m$ is:

$T(r,t) = [T_s^{x+1} + \frac{u_c''' R^2 (x+1) T_s^x}{3k_s(m+2)}(1 - (\frac{r}{R})^{2+m})]^{\frac{1}{x+1}}$

$+[\frac{2}{R}\sum_{n=1}^{\infty} A_n \frac{\sin(\frac{n\pi r}{R})}{r} \exp(-\alpha(\frac{n\pi}{R})^2 t)]^{\frac{1}{x+1}}$,

where α is constant, and where:

$A_n = \int_0^R T_i - \{[T_s^{x+1} + \frac{u_c''' R^2 (x+1) T_s^x}{3k_s(m+2)}(1 - (\frac{r}{R})^{2+m})]^{\frac{1}{x+1}}\}^{x+1} r \sin(\frac{n\pi r}{R}) dr.$

Check for case $u_c''' = 0$:

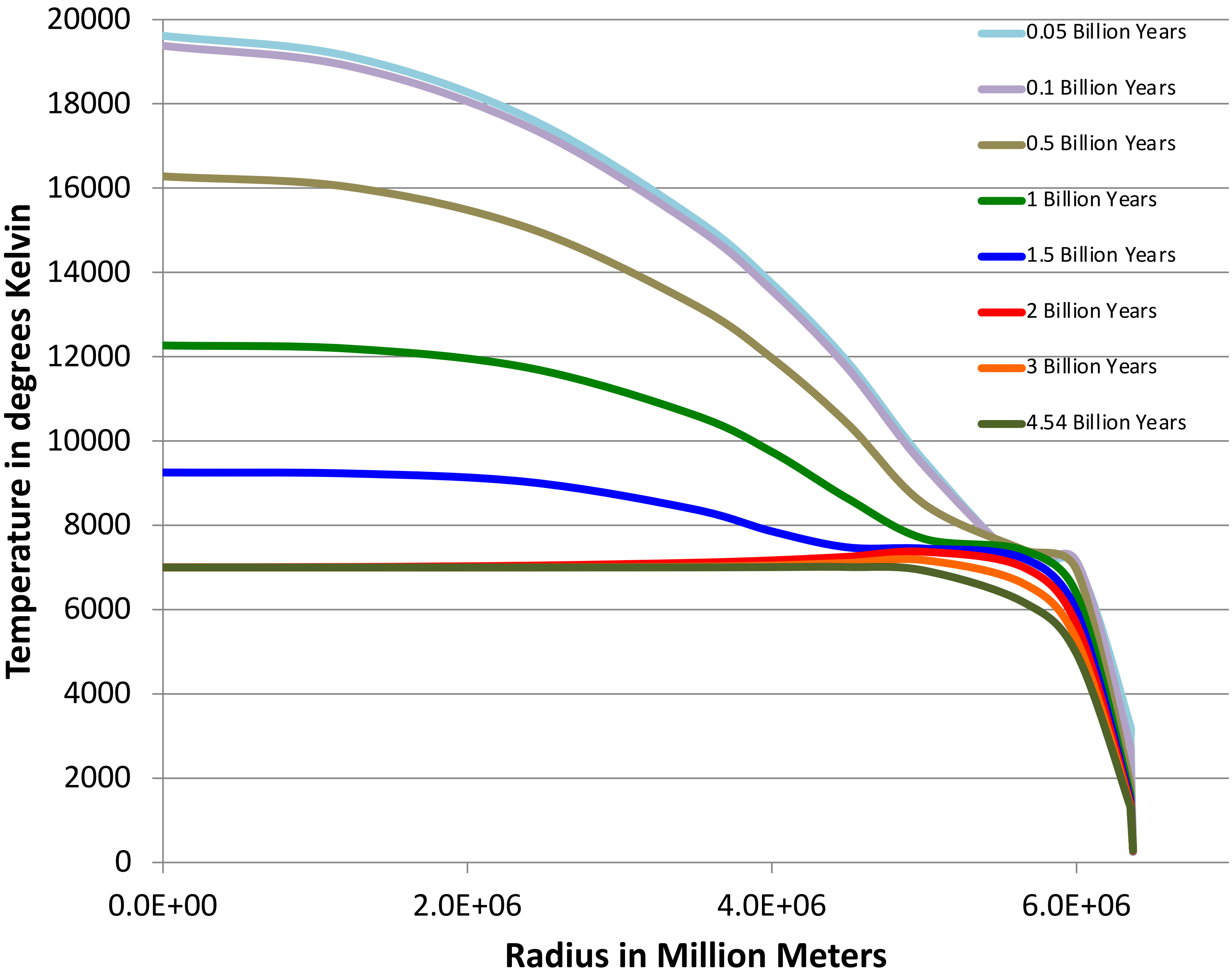
$T(r,t) = T_s + [\frac{2}{R}\sum_{n=1}^{\infty} A_n \frac{\sin(\frac{n\pi r}{R})}{r} \exp(-\alpha(\frac{n\pi}{R})^2 t)]^{\frac{1}{x+1}}$,

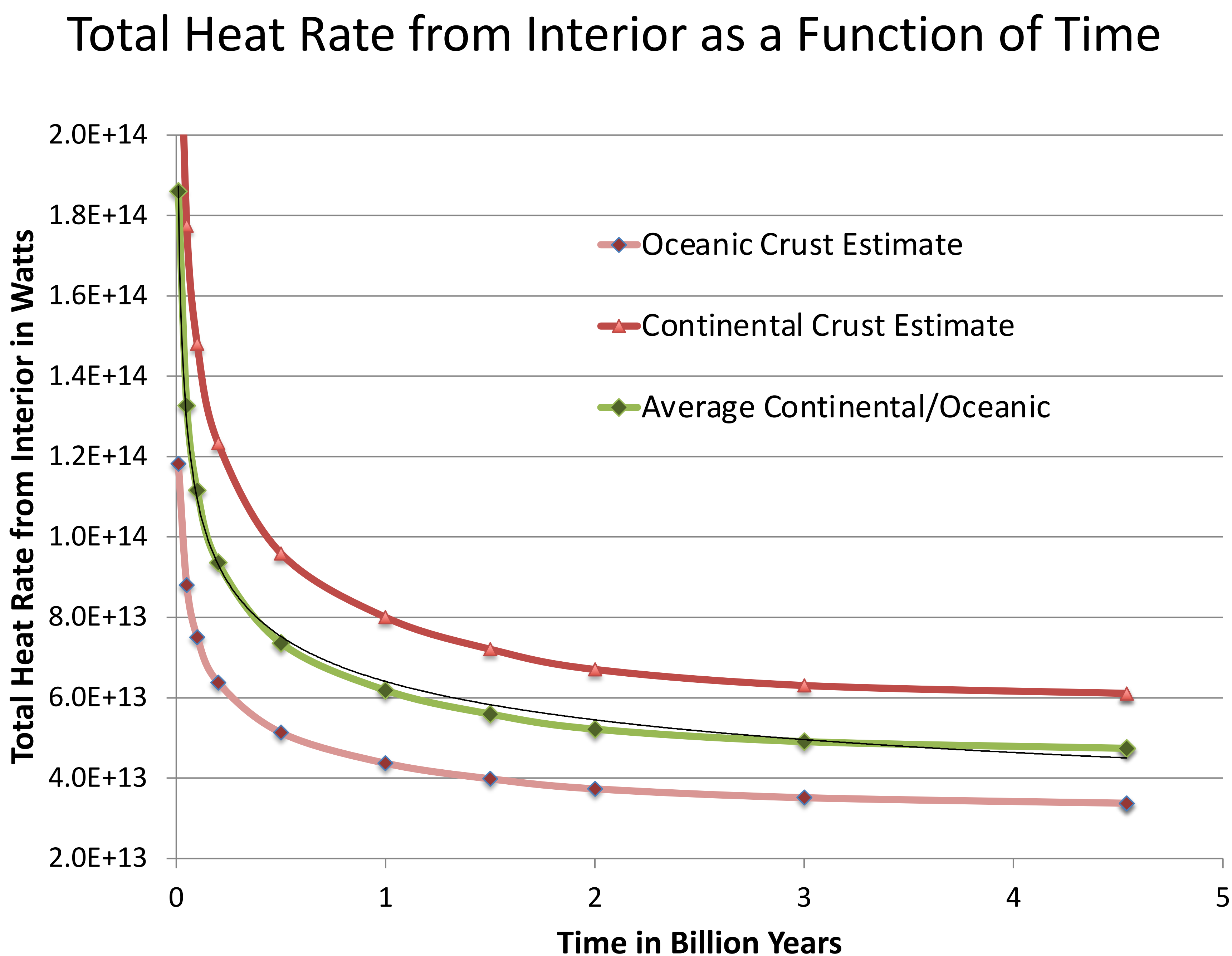
where $A_n = \int_0^R (T_i - T_s)^{x+1} r \sin(\frac{n\pi r}{R}) dr.$

For constant $(T_i - T_s)$ independent of r, $A_n = (T_i - T_s)^{x+1}(\frac{-R^2(-1)^n}{n\pi})$,

$T(r,t) = T_s + (T_i - T_s)[2R\sum_{n=1}^{\infty} \frac{-(-1)^n}{n\pi} \frac{\sin(\frac{n\pi r}{R})}{r} \exp(-\alpha(\frac{n\pi}{R})^2 t)]^{\frac{1}{x+1}}.$

Distributed Earth Model: Variable k (T), u'''(r) and T_s (t)




Total Heat Rate from Interior as a Function of Time
Total Heat Rate from Interior in Watts
2.0E+14
1.8E+14
1.6E+14
1.4E+14
1.2E+14
1.0E+14
8.0E+13
6.0E+13
4.0E+13
2.0E+13
0
1
2
3
4
5
Time in Billion Years
Oceanic Crust Estimate
Continental Crust Estimate
Average Continental/Oceanic

Future Targets

- Explore analytical solutions further by incorporating an "accretion / initial thermal condition." Then predict for each time step Earth's interior thermal distribution based on previous (earlier time) estimates of developing thermal profile.

- Incorporate analytically the thermal effects of core solidification and viscous dissipation.

- Explore supercomputing simulations, first with pure conduction model, then with advection of fluid iron core, finally with mantle advection.

This past decade's proposal to "seed" with small particulates Earth's atmosphere (goal to reduce incoming solar radiation and Earth's global average surface temperature) is ***flawed*** for at least two reasons:

1. As evidenced in ice cores and paleogeophysics research, volcanic emissions preceded historical ice ages on Earth.
2. Means of "thermal control" are substantially reduced with dispersion of particulates into Earth's atmosphere: Second Law of Thermodynamics precludes energy efficient "re-collection" of dispersed particulates.

Alternate: Proposed Science and Engineering collaboration

Analyze, design, test and build solar shields / shades, deployable at intermediate spatial location(s) between the sun and Earth, particularly the inner Lagrange point L1, to reduce incoming solar radiation by approximately 1%. Build public confidence using Martian surface as testbed platform.