Thermal Distribution in Earth’s Interior

ME 531 Graduate Conduction Heat Transfer
at the University of Washington
Mechanical Engineering Dept.

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4.54 Billion Year Earth History

- **4550 Ma:** Formation of the Earth
- **Hominins**
- **Mammals**
- **Land plants**
- **Animals**
- **Multicellular life**
- **Eukaryotes**
- **Prokaryotes**
- **Hadean**
- **Archean**
- **Proterozoic**
- **Paleozoic**
- **Mesozoic**
- **Cenozoic**

- **4527 Ma:** Formation of the Moon
- **4.6 Ga**
- **4 Ga**
- **3 Ga**
- **2.5 Ga**
- **2 Ga**
- **1 Ga**
- **541 Ma**
- **252 Ma**
- **66 Ma**

- **2 Ma:** First Hominins
- **230-66 Ma:** Non-avian dinosaurs
- **c. 380 Ma:** First vertebrate land animals
- **c. 540 Ma:** Cambrian explosion
- **c. 650–635 Ma:** Marinoan Glaciation (Snowball Earth event)
- **c. 716–660 Ma:** Sturtian Glaciation (Snowball Earth event)

- **c. 540 Ma:** Cenozoic
- **c. 540 Ma:** Cambrian explosion
- **c. 4000 Ma:** End of the Late Heavy Bombardment; first life
- **c. 3200 Ma:** Earliest start of photosynthesis
- **c. 2300 Ma:** First major increase in atmospheric oxygen levels; first Snowball Earth event (Huronian glaciation)
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- **230-66 Ma:** Non-avian dinosaurs
Figure 1. Evolution of solar luminosity over the four geologic eons for the standard solar model described in Bahcall et al. [2001] (solid line) and according to the approximation formula [Gough, 1981] (dashed line) given in equation (1).
Consensus on Evolution of Solar Luminosity

“The gradual increase in luminosity during the core hydrogen burning phase of evolution of a star is an inevitable consequence of Newtonian physics and the functional dependence of the thermonuclear reaction rates on density, temperature and composition.” [Gough, 1981, p. 28]
Liquid Water Evidence in Archean Eon

- “Telltale signs of liquid water include pillow lavas that are formed when lava extrudes under water, ripple marks resulting from sediment deposition under the influence of waves, and mud cracks.”
- “. . evidence for microbial life in the Archean derived from microfossils or stromatolites / microbial mats in rocks of ages between 2.5 and 3.5 Gyr [Barghoorn and Schopf, 1966; Altermann and Kazmierczak, 2003; Schopf, 2006]”
- “. . no evidence for widespread glaciations during the entire Archean [Lowe, 1980; Walker, 1982; Walker et al, 1983; Fowler et al, 2002; Eriksson et al, 2004; Benn et al, 2006]”
“Paradox” from Global Energy Balance

- Earth’s global average surface temperature is ~ 288 K, governed by current normal solar incidence (1360 W/m\(^2\)) at the top of the atmosphere.
- **Without** thermal effects of the atmosphere, Earth’s global average surface temperature would instead be ~ 278 K. (“Thermal blanket” adds surface average ~ 10 K.)
- Without atmospheric “thermal blanketing,” Earth’s global average surface temperature would have been ~ 259K, at 75% of current solar luminosity.
- At 80% of current solar luminosity, Earth’s global average surface temperature would have been ~ 263K, still well below H\(_2\)O freezing at standard atmospheric pressure.
Why Was Archean Earth Not Frozen?

✓ “Observations of other cool stars show that they lose most of their mass during first 0.1 Gyr. Most importantly, the observed solar analogs exhibit considerably lower cumulative mass-loss rates than required to offset the low luminosity of the early Sun [Minton and Malhotra, 2007].”

✓ Enhanced green house effects of Ammonia, Methane, Carbon Dioxide, other greenhouse gases checked.

✓ Cloud effects: “... it appears unlikely that any cloud effect alone can resolve the faint young Sun problem...”

✓ Rotation and Obliquity effects checked.

✓ Ocean Salinity and Tide effects checked.
Surface temperature distribution on liquid water Earth, with no atmosphere, Thermal Radiogenic Production 50 TerraWatts
Predicted Temperature Distribution on Liquid Water Surface, Non-rotating Earth without Atmosphere

Temperature (°C)

Theta in degrees (normal incidence at Theta = 0)
Earth’s Global **Average** Surface Temperature as a Function of Time

![Graph showing the average surface temperature of Earth over time. The x-axis represents time in billion years, ranging from 0 to 5. The y-axis represents temperature in degrees Kelvin, ranging from 230 to 300. The graph includes two data series: Blanketing Atmosphere (diamonds) and Radiation Balance (squares).]
With Early Solar Correction

Predicted Earth Average Surface Temperature (K)

Heat Rate from Earth Interior (Watts): Log Scale

From Earth Interior
Incoming Solar

Time in Years: Log Scale

Time in Years

1.0E+00 1.0E+09 2.0E+09 3.0E+09 4.0E+09

2.0E+13
2.0E+14
2.0E+15
2.0E+16
2.0E+17

1.0E+00 1.0E+09 2.0E+09 3.0E+09 4.0E+09

230
240
250
260
270
280

2.0E+00 1.0E+04 1.0E+06 1.0E+08

230
240
250
260
270
280
Intermediate (Important) Conclusions

For the purpose of modeling Earth’s interior thermal distribution, it is reasonable to treat its surface temperature $T_{\text{surface}}$ as “constant” over a time period of 0.30 Billion to 4.54 Billion Years.

$T_{\text{surface}} \sim 280$ K

Earth’s average surface temperature is indeed affected by its atmosphere, but far and away the most dominant influence is solar flux.
Earth’s Interior Challenges

Thermal conductivities, densities and specific heats vary spatially with material composition and as a function of temperature.

+ Thermal diffusivities vary far less

Radiogenic heat production is a function of spatial coordinates and time.

+ Current source estimates ~20-25 TW likely to be close; estimates of historic radiogenic production reasonable.

Total heat rate from interior based on 38,000 measurements, reported as 47 +/- 2 TeraWatts.
Why attempt analytical modeling? (Rather than proceeding full force with simulation?)

Disparate physical scales:
- Average Earth radius ~ 6371 km
- Conduction at kinetic molecular collision scale

Disparate time scales:
- Earth age ~ 4.54 Billion Years
- Simulate conduction with time steps ~ seconds
Conduction within solid spherical Earth, where \( r = \) radius, \( \theta = \) latitudinal angle, and \( \varphi = \) circumferential angle. The outer radius, \( R = 6371 \) km. The current radiogenic production, \( u''' \forall \approx 20 \) TW decreasing from \( u''' \forall \approx 100 \) TW @ accretion.

For uniform conductivity \( k \):
\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \varphi^2} + \frac{u'''}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}.
\]

For variable thermal conductivity \( k \):
\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \varphi} \left( k \frac{\partial T}{\partial \varphi} \right) + u''' = \rho c \frac{\partial T}{\partial t}.
\]

Boundary conditions? \( T|_{\varphi} = T|_{\varphi+2\pi} \), and \( \frac{\partial T}{\partial \varphi}|_{\varphi} = \frac{\partial T}{\partial \varphi}|_{\varphi+2\pi} \).

Likewise in \( \theta \), continuity in temperature and thermal gradient \( \frac{\partial T}{\partial \theta} \).

Largest thermal changes in radial \( r \).

Neglecting \( \frac{\partial T}{\partial \varphi} \) and \( \frac{\partial T}{\partial \theta} \), the boundary conditions in \( r \):
\[
\frac{\partial T}{\partial r}|_{r=0} = 0, \text{ and } T|_{r=R} = T_{\text{surface}} \approx 280 \text{K}.
\]

Solve \( \frac{1}{r^2} \frac{\partial}{\partial r} \left( kr^2 \frac{\partial T}{\partial r} \right) + u''' = \rho c \frac{\partial T}{\partial t} \).
Earth’s “steady” Interior Temperatures
for u’’’*Volume = 41 TW

Temperature in degrees Kelvin
Radial Coordinate in meters

Uniform k=24W/mK
Variable K
Is the Earth close at all to steady state?
The answer is NO: Prediction for $k = 24 \text{ W/(m K)}$, $u'''V = 41 \text{ TW}$
Total Heat Rate from Interior, $k = 24 \text{ W/(m K)}$, $u'''V = 41 \text{ TW}$
Two most easily identified problems:

• The Earth’s interior thermal conductivity is not uniform!

• The Earth’s thermogenic production is spatially and temporally variable!
Thermal Radiogenic Variation in Time

Radiogenic Production in Watts

Time in Billion Years

From Potassium, Uranium, Thorium

Time averaged cumulative
Estimated \( u'''' \) as a Function of Radial Coordinate and Time

Thermogenic Production \( u'''' \) (W/m\(^3\))

- Accretion
- 0.05 Billion Years
- 0.1 Billion Years
- 0.5 Billion Years
- 1 Billion Years
- 3 Billion Years
- 4.54 Billion Years

Radial Coordinate (m)

<table>
<thead>
<tr>
<th>Time (Billion Years)</th>
<th>Radial Coordinate (m)</th>
</tr>
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<tr>
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<tr>
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<td>0.00 - 6.00</td>
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<tr>
<td>4.54</td>
<td>0.00 - 6.00</td>
</tr>
</tbody>
</table>
Varying index $m$: $3u''' = 3u_s''' (r/R)^m = (3+m)u_c'''(r/R)^m$
With uniform conductivity \( k \): 
\[
T(r, t) = T_{\text{surface}} + \frac{u'''}{6k} (R^2 - r^2) + \sum_{n=1}^{\infty} \left\{ \frac{2R(-1)^n}{n\pi} \left[ \frac{u'''}{k(n\pi)^2} - (T_i - T_{\text{surface}}) \right] \frac{\sin \left( \frac{n\pi r}{R} \right)}{r} \exp \left( -\frac{k(n\pi)^2}{\rho c R^2} t \right) \right\},
\]
for initial uniform temperature \( T(r, t = 0) = T_i \).

For variable thermal conductivity \( k = k_s \frac{T}{T_s} \):
\[
T(r, t) = \sqrt{T_s^2 + T_s \frac{u'''}{3k_s} (R^2 - r^2)} + \sqrt{2 \sum_{n=1}^{\infty} T_s B_n \frac{\sin \left( \frac{n\pi r}{R} \right)}{r} \exp \left( -\alpha \left( \frac{n\pi}{R} \right)^2 t \right)},
\]
where thermal diffusivity \( \alpha = \frac{k}{\rho c} \) is constant, and where:
\[
T_s B_n = \frac{T_s R^3 u'''}{3k_s n\pi} \left( 1 - \frac{6}{(n\pi)^2} \right) - \frac{R(-1)^n}{n\pi} \left( T_i^2 + T_s^2 + T_s R^2 \frac{u'''}{3k_s} \right) - \frac{2T_i}{R} \int_0^R r \sin \left( \frac{n\pi r}{R} \right) \sqrt{T_s^2 + T_s \frac{u'''}{3k_s} (R^2 - r^2)} \, dr.
\]

Analytical solution for \( k = k_s \left( \frac{T}{T_s} \right)^x \), and \( u''' = u'_s \left( \frac{r}{R} \right)^m \)? Yes!

“Steady state” solution \( T(r) = \left[ T_s^{x+1} + \frac{u'''}{3k_s n\pi} \left( 1 - \left( \frac{r}{R} \right)^2 \right)^{m+2} \right] \frac{1}{x+1} \),
where \( u'' = u'_s \frac{3}{3+m} \), and \( u''' \bigg|_{r=R} = u'_s \).
Transient 1-D solution for $k = k_s \left( \frac{T}{T_s} \right)^x$ and $u''' = u''_s \left( \frac{r}{R} \right)^m$ is:

$$T(r, t) = \left[ T_s^{x+1} + \frac{u'''_s R^2 (x+1) T_s^x}{3 k_s (m+2)} (1 - \left( \frac{r}{R} \right)^{2+m}) \right] \frac{1}{x+1}$$

$$+ \left[ \frac{2}{R} \sum_{n=1}^\infty A_n \frac{\sin \left( \frac{n \pi r}{R} \right)}{n \pi} \exp \left( -\alpha \left( \frac{n \pi}{R} \right)^2 t \right) \right] \frac{1}{x+1},$$

where $\alpha$ is constant, and where:

$$A_n = \int_0^R T_i - \left\{ \left[ T_s^{x+1} + \frac{u'''_s R^2 (x+1) T_s^x}{3 k_s (m+2)} (1 - \left( \frac{r}{R} \right)^{2+m}) \right] \frac{1}{x+1} \right\} x+1 r \sin \left( \frac{n \pi r}{R} \right) dr.$$

Check for case $u'''_s = 0$:

$$T(r, t) = T_s + \left[ \frac{2}{R} \sum_{n=1}^\infty A_n \frac{\sin \left( \frac{n \pi r}{R} \right)}{n \pi} \exp \left( -\alpha \left( \frac{n \pi}{R} \right)^2 t \right) \right] \frac{1}{x+1},$$

where $A_n = \int_0^R (T_i - T_s)^{x+1} r \sin \left( \frac{n \pi r}{R} \right) dr.$

For constant $(T_i - T_s)$ independent of $r$, $A_n = (T_i - T_s)^{x+1} \frac{(-1)^n}{n \pi} \frac{\sin \left( \frac{n \pi r}{R} \right)}{n \pi} \exp(-\alpha \left( \frac{n \pi}{R} \right)^2 t) \frac{1}{x+1}.$
Distributed Earth Model: Variable $k(T)$, $u'''(r)$ and $T_s(t)$

Temperature in degrees Kelvin

Radius in Million Meters

- 0.05 Billion Years
- 0.1 Billion Years
- 0.5 Billion Years
- 1 Billion Years
- 1.5 Billion Years
- 2 Billion Years
- 3 Billion Years
- 4.54 Billion Years
Future Targets

- Explore analytical solutions further by incorporating an “accretion / initial thermal condition.” Then predict for each time step Earth’s interior thermal distribution based on previous (earlier time) estimates of developing thermal profile.

- Incorporate analytically the thermal effects of core solidification and viscous dissipation.

- Explore supercomputing simulations, first with pure conduction model, then with advection of fluid iron core, finally with mantle advection.
This past decade’s proposal to “seed” with small particulates Earth’s atmosphere (goal to reduce incoming solar radiation and Earth’s global average surface temperature) is *flawed* for at least two reasons:

1. As evidenced in ice cores and paleogeophysics research, volcanic emissions preceded historical ice ages on Earth.

**Alternate: Proposed Science and Engineering collaboration**

Analyze, design, test and build solar shields / shades, deployable at intermediate spatial location(s) between the sun and Earth, particularly the inner Lagrange point L1, to reduce incoming solar radiation by approximately 1%. Build public confidence using Martian surface as testbed platform.